Structuring and Restructuring Sovereign
Debt: The Role of a Bankruptcy Regime
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# IMF Working Paper 

Research Department

# Structuring and Restructuring Sovereign Debt: The Role of a Bankruptcy Regime 

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#### Abstract

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In an environment characterized by weak contractual enforcement, sovereign lenders can enhance the likelihood of repayment by making their claims more difficult to restructure ex post. We show however, that competition for repayment among lenders may result in a sovereign debt that is excessively difficult to restructure in equilibrium. This inefficiency may be alleviated by a suitably designed bankruptcy regime that facilitates debt restructuring.

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## I. Introduction

The composition of sovereign debt and how it affects debt restructuring negotiations in the event of financial distress has become a central policy issue in recent years. There are two major reasons why the spotlight has been turned on this question. First, the change in the IMF's policy orientation towards sovereign debt crises, with a proposed greater weight on "private sector involvement" (Rey Report, G-10, 1996), has brought up the question of how easy it actually is to get 'the private sector involved'; that is, how easy it is to get private debt-holders to agree to a debt restructuring. Second, the experience with several recent debt restructuring episodes-some of which were followed by defaults and by private litigation to recover debt payments-have raised concerns that the uncoordinated efforts of dispersed debt-holders to renegotiate sovereign debt obligations were likely to lead to substantial delays and other inefficiencies.

These concerns have led a number of prominent commentators, a majority of G-7 countries, and the IMF to advocate ex-post policy interventions to facilitate debt restructuring. A culmination point for the calls for reform was reached when the IMF's Anne Krueger put forward the idea of a sovereign debt restructuring mechanism (SDRM) inspired by the U.S. corporate bankruptcy reorganization law under Chapter 11 of the 1978 Bankruptcy Act (Krueger, 2002). ${ }^{1}$ The ensuing policy debate has, however, left many commentators wondering why, in the first place, sovereign debt had been structured to make it difficult to renegotiate, and why the structure of sovereign debt had evolved over the past decade or so towards a greater share of sovereign bond issues and greater dispersion of ownership of sovereign bonds. This paper is concerned with precisely these issues. Its starting points are the questions:

1) why would a forward looking sovereign want to design a sovereign debt structure that is difficult to restructure?
2) where are the contractual failures between the borrower and lenders that justify an ex-post policy intervention to facilitate debt restructuring?

Several commentators (Dooley, 2000; Shleifer, 2003) have argued that due to the sovereign's incentive to repudiate its debts (the well known willingness-to-pay problem) it may be exante efficient to structure sovereign debt to make it difficult to renegotiate ex-post. A policy intervention that aims to reduce these restructuring costs, while improving ex-post efficiency,

[^0]might thus undermine ex-ante efficiency. Such a policy would have the effect of raising the cost of borrowing and would result in a reduction of lending to emerging market countries. ${ }^{2}$

Our paper builds on this very idea that debts that are more difficult to restructure are less vulnerable to repudiation, but it stops short of concluding that sovereign debt that is difficult to restructure is necessarily ex-ante efficient. We add to the theme that lenders seek protection against a generalized default by a sovereign, the idea that individual lenders also seek to protect themselves individually against selective defaults by the sovereign on a subset of its debts. Thus, by attempting to divert a selective default onto other debts, individual lenders may end up providing too much debt that is difficult to restructure. Just as a burglary alarm may be an individually optimal protection against break-ins for an individual house owner (by inducing prospective burglars to target other houses without such an alarm system), collectively, having all houses equipped with an alarm could well be self-defeating and inefficient.

By lending in the form of debt that is hard to restructure, individual lenders are able to effectively make their debts more senior to other debts that are easier to restructure and, therefore, more likely to be selected for a default by the sovereign ex-post. Or, put differently, with each debt issue, the sovereign may attempt to lower the cost of borrowing by committing to high future restructuring costs of that particular issue and thus providing a form of seniority to that issue. This de facto seniority can be obtained in various ways, for example by lifting sovereign immunity, by widely dispersing the debt and insisting on a unanimity requirement for restructuring the debt, by lowering the maturity of the debt, by denominating the debt in dollars, or by inserting acceleration clauses. Thus, a form of Gresham law for sovereign debt may arise-where bad debt structures which are hard to restructure tend to crowd out good debts that are easier to renegotiate.

Our paper argues that there is, therefore, a role for policy intervention in sovereign lending that would improve both ex-ante and ex-post efficiency. This policy intervention should take the general form of facilitating the restructuring or hard debt. Thus, our theory has some implications for the reforms of the international financial architecture that have been discussed in recent debates, and in particular the desirability of a bankruptcy regime for sovereigns. We argue that because of the competition between lenders to deflect a selective default, sovereign debt might be excessively hard to restructure in equilibrium even from an ex ante perspective. A bankruptcy regime for sovereigns could then mitigate this inefficiency by facilitating debt restructuring in a sovereign debt crisis.

[^1]In our model, the contractual approach to sovereign debt restructuring recently endorsed by the G-10, ${ }^{3}$ which is limited to moral suasion over issuers to introduce majority-rule clauses for the restructuring of debt in bond issues (so-called collective action clauses or CACs) does not work. ${ }^{4}$ As we show, efficiency cannot be achieved by leaving sovereign borrowers free to include or not renegotiation-friendly clauses in their debt. In equilibrium, the adoption of such clauses will be inefficiently low. However, a policy that encourages the adoption of such clauses through a system of taxes or subsidies or other arm-twisting (as advocated by Eichengreen, 2003, or Kenen, 2001), or by making their use mandatory, could achieve the same effect as a restructuring under a bankruptcy regime.

Although our analysis provides support for a bankruptcy regime or some form of mandated or subsidized collective action clauses, we also emphasize that such an intervention may easily be welfare-reducing if it is not carefully designed. Indeed, it could undermine sovereign debt markets if it gives lenders too little bargaining power in a renegotiation.

Our paper contributes to the literature on sovereign debt default and restructuring. A number of authors have emphasized the importance of selective default in sovereign debt. Dooley (2000) and Kenen (2001) for example emphasize the conflict between official and private lenders in the competition for repayment. Tirole (2002, chapter 4) discusses the contracting externalities arising from selective default and mentions seniority as a possible solution to this problem. As we document in Section II, practitioners also pay a great deal of attention to the implicit seniority status of the different types of sovereign debt. However, although commentators and practitioners are aware of the issue, the implications of selective default on sovereign debt have to our knowledge not been explored systematically before.

The paper is structured as follows. Section II reviews some stylized facts on sovereign debt that motivate the theoretical analysis in the rest of the paper. Section III lays out the model and basic assumptions. Section IV characterizes the socially efficient debt structure. Section V analyzes equilibrium debt structures. Section VI discusses public policy implications and Section VI concludes.

## II. Evidence on Selective Defaults

This section presents evidence suggesting that there is an implicit seniority structure for sovereign debt, and that this structure is related to the perceived difficulty with which debt can be restructured. The implicit seniority in sovereign debt is an understudied topic, on which there has been little empirical and theoretical research. We will present a few facts as

[^2]well as market commentaries that suggests that seniority is a real issue for sovereign debt structuring and restructuring-see more detailed discussions of the evidence in Roubini and Setser (2004), Sturzenegger and Zettelmeyer (2006), and Zettelmeyer (2003).

The de facto seniority structure of sovereign debt is, for one thing, apparent from the different treatment of different classes of creditors in a default. The differential treatment of claims has been a characteristic of most debt restructurings that have taken place over the last 25 years (beginning with the debt renegotiations and write-downs of the 1980s and the Brady plan, and continuing with the more recent debt restructurings in Russia, Ukraine, Pakistan, Ecuador, and Uruguay). This differential treatment has often involved more favorable terms for some subclasses of private claims. Thus, for example, the Brady deals that settled the debt crises of the 1980s restructured bank loans but not international bonds (Merrill Lynch, 1995).

Over the past two decades, the composition of sovereign debt has shifted away from syndicated bank loans, which were the dominant form of lending in the 1970s and 1980s, toward bond finance. While there is no single cause that explains this change in composition, one reason may have been the perception, following the debt crises of the 1980s and the Brady deals, that syndicated bank loans were too easy to restructure. In valuing the new bond issues, at least some lenders have factored in a lower risk of restructuring of international bonds. To the extent that these bond issues were widely dispersed, they were perceived to be more difficult to restructure, and therefore less likely to be restructured in a debt crisis:

> There are several things that make international bonds much harder to restructure than loans. First, they typically involve many more investors than do loans, even syndicated loans. Second, they may be in bearer form so investors may be untraceable. (Euromoney, ${ }^{5}$ October 1999).

The recent debt crisis and default of Argentina has highlighted just how difficult comprehensive debt restructuring negotiations can be, when they involve hundreds of thousand different bondholders with a wide variety of objectives. ${ }^{6}$

During most of the 1990s the differential treatment of sovereign claims has followed a pattern that is consistent with an implicit seniority of international bonds over international bank loans. A total of 93 sovereigns have defaulted on their syndicated bank loans since

[^3]1975, including 20 that had bonds outstanding at the same time as their bank loans were in default. Yet, only 9 out of these 20 sovereigns also defaulted on their bonds, and the others serviced them in full (Standard and Poor's, 2003).

The restructuring of Russian sovereign debt (August 1998-August 2000) is typical of this pattern. Domestic debt and Soviet era London and Paris Club debts have been restructured (with international bank creditors accepting a debt exchange involving a 40 percent reduction in the present value of their claims), while Eurobonds have been left untouched. Market participants have viewed this latest Russian debt restructuring episode as further corroboration of sovereigns' tendency to treat creditors differently according to their power of nuisance.

Market participants were also well aware that such behavior resulted in an implicit seniority structure affecting the pricing and valuation of debt:

It is that implicit seniority which, in part, explains why bonds have become such favoured instruments for countries raising debt in recent years. (Euromoney, October 1999, p. 50)

The majority of governments treated bonds as being effectively senior to bank loans, and they did so with the tacit consent of bank creditors. (Standard and Poor's, 2003)

To summarize, the evidence points to the following stylized facts that our theory will attempt to capture and explain:

- Sovereigns do not default in the same way on different classes of debt instruments and this selectivity generates an implicit seniority between debt classes.
- Seniority seems related to structural features that make some types of sovereign debt easier to renegotiate than others.
- International investors are aware of this implicit seniority structure and pay close attention to potential shifts in its determinants.
- The composition of international sovereign debt has shifted toward the class of instruments that was perceived as senior during the 1990s.


## III. The Model: Assumptions

We consider a small open economy over two periods with a single homogenous good that can be consumed or invested. The representative resident of this economy may raise funds from the rest of the world by issuing (sovereign) debt in the first period $(t=1)$ to be repaid in the next period $(t=2)$. The funds raised in the first period can be used for consumption or investment purposes.

To keep the analysis as tractable as possible we specify the following simple form for the utility function of the representative resident:

$$
\begin{aligned}
& U_{2}=V(g)+c, \\
& U_{1}=E_{1}\left(U_{2}\right),
\end{aligned}
$$

where $g$ is the level of public expenditure in period 1 and $c$ is private consumption in period 2. The sovereign is assumed to act on behalf of the representative resident and maximizes her welfare.

The representative resident receives an exogenous stochastic endowment $y$ in period 2 , which is distributed according to the probability distribution function $f($.$) over the support$ $[0, \bar{y}]$, where $\bar{y}<\infty$. Again for simplicity we shall assume that there is no taxable output in period 1 so that the government finances $g$ entirely by borrowing from foreign lenders. We shall assume that $g$ is exogenously given and such that $0<g<E[y]$, where $E[y]$ denotes the expected endowment in period $2 .{ }^{7}$

In reality most sovereigns' borrowing needs are such that they have no choice but to borrow from multiple lenders. When a sovereign borrows from different lenders the issue of strategic default is more complex, as the sovereign can choose to selectively default on some of the lenders and not others. Accordingly, any individual lender will be concerned not only about the risk of a full default by the sovereign, but also about the relative risk of a selective default on its own debt. An individual lender can guard himself to some extent against the risk of a selective default by lending through a debt instrument that is difficult to restructure. Thus, to allow for multiple lenders, as well as different types of debts in terms of how difficult they are to restructure, we shall model the sovereign debt lending game as follows.

## A. Lending Game

In period 1 there is a continuum of atomistic lenders (indexed by $i$ ) from which the sovereign can borrow. Each of these lenders is able to lend $g$, so that the sovereign must

[^4]borrow from a subset of mass 1 of lenders. The total mass of lenders is equal to $1+\varphi$, where $\varphi>0$ is a small number. This ensures that perfect competition prevails and lenders do not extract any rent. The lenders have access to a zero-return storage technology.

The lending game can be viewed as a general (common agency) contracting game between a principal (the sovereign) and multiple agents (the lenders) as, for example, in Bernheim and Whinston (1986a,b), Hart and Tirole (1990), or Segal (1999). Specifically, we follow Segal (1999) by letting lenders participate in a bidding game following the sovereign's announcement of a fund raising goal of $g$. Lenders move first by each simultaneously making a bid. The sovereign then decides which bids to accept.

Lenders can provide two types of debt to the sovereign: renegotiable debt and nonrenegotiable debt. At the bidding stage of the game each lender decides which type of debt to offer and under what terms. Thus, the action of lender $i$ is characterized by the pair $(\theta, d(i))$, where $\theta=r, n$ is an indicator variable for the type of debt offered and $d(i)$ is the period-2 debt repayment requested by the lender in return for a loan of $g$.

In the second stage of the bidding game the sovereign chooses which bids to accept. At the end of the bidding game the sovereign is thus indebted with two classes of creditors: r -creditors (the holders of renegotiable debt) and n-creditors (the holders of non-renegotiable debt).

The noncooperative nature of the game reflects the idea that it is difficult for lenders to coordinate themselves or be coordinated by the sovereign. Given that there is an excess supply of lenders, the latter attempt to win the lending contest by offering the most attractive terms to the government. Therefore, one should expect an equilibrium outcome where the sovereign receives all the surplus from the lending relationship.

## B. Repayment Game

We shall denote by $N_{r}$ and $N_{n}$ the respective mass of r-creditors and n-creditors. We look at symmetric equilibria in which all creditors of a given type $\theta=r, n$ make the same bid $d_{\theta}$, so that the sovereign's total repayments of renegotiable debt and non-renegotiable debt that comes due in period 2 is

$$
d_{r} N_{r}+d_{n} N_{n} \equiv D_{r}+D_{n} .
$$

The promise to repay $\left(D_{r}+D_{n}\right)$ is credible only if it is in the sovereign's interest to repay its debt obligations ex post. We follow the sovereign debt literature by assuming that the sovereign repays its debts only as a way of avoiding a costly default. As in Sachs and Cohen
(1982) and Obstfeld and Rogoff (1996), we model the cost of default as a proportional output loss, $\gamma y .{ }^{8}$ We interpret this cost as a sanction imposed by creditors on the defaulting sovereign (see Bulow and Rogoff, 1989, for a discussion of such sanctions). ${ }^{9}$ To simplify the algebra, and without loss of generality, we assume that $\gamma=1$, so that the creditors can destroy all the resources of a defaulting sovereign.

For simplicity we assume that renegotiable debt can be renegotiated at no cost, but that nonrenegotiable debt is impossible to renegotiate since these debts are too widely dispersed and since a unanimous agreement is required to renegotiate the debt. ${ }^{10}$ For example, one can think of the renegotiable debt as syndicated bank loans, and non-renegotiable debt as bonds held by a large number of dispersed bondholders.

The sovereign is always better off repaying the n -creditors than losing all the domestic output because of the sanctions. Thus a full default (on both types of creditors) will occur only if the sovereign is unable to repay the n -creditors,

$$
y<D_{n} .
$$

We shall assume that in a selective default r-creditors receive a fraction $\omega \in[0,1]$ of the net surplus from renegotiation $y-D_{n}$ (net after repayment of the n-creditors). It follows that the sovereign chooses a selective default over full repayment if,

$$
y-D_{n}-\omega\left(y-D_{n}\right)>y-D_{n}-D_{r}
$$

or if,

$$
y<D_{n}+\frac{D_{r}}{\omega} .
$$

[^5]To summarize the payoffs of the different players in the selective default game are given in the table below.

|  | Full Default | Selective Default | Full Repayment |
| :--- | :--- | :--- | :--- |
|  | $y<D_{n}$ | $D_{n} \leq y<D_{n}+\frac{D_{r}}{\omega}$ | $D_{n}+\frac{D_{r}}{\omega} \leq y$ |
| Sovereign | 0 | $(1-\omega)\left(y-D_{n}\right)$ | $y-D_{r}-D_{n}$ |
| r-creditors | 0 | $\omega\left(y-D_{n}\right)$ | $D_{r}$ |
| n-creditors | 0 | $D_{n}$ | $D_{n}$ |

And we obtain the following result characterizing ex post equilibrium default:
Proposition 1. The sovereign's debt repayment strategy is as follows:
(i) if $y \geq D_{n}+\frac{D_{r}}{\omega}$, the sovereign fully repays both types of debt;
(ii) if $D_{n} \leq y<D_{n}+\frac{D_{r}}{\omega}$, the sovereign fully repays the non-renegotiable debt and agrees on a reduction of the renegotiable debt to $\omega\left(y-D_{n}\right)$ with the $r$-creditors;
(ii) if $y<D_{n}$, the sovereign defaults on both types of debt.

Proof. See the discussion above.

This proposition highlights the notion that non-renegotiable debt is effectively senior to renegotiable debt. In the event of a selective default, the allocation of the repayments between r-creditors and n-creditors is the same as if the latter enjoyed strict seniority over the former. Because of this effective seniority, n-creditors have a larger expected recovery ratio than r-creditors, so that the interest rate spread should be lower on non-renegotiable debt than on renegotiable debt.

## IV. Optimal Debt Structure

As a benchmark, we begin by characterizing the debt structure chosen by a social planner, subject to the lenders' participation constraints. That constraint is given by:

$$
\begin{equation*}
V\left(D_{r}, D_{n}\right) \geq g, \tag{1}
\end{equation*}
$$

where $V\left(D_{r}, D_{n}\right)$, the lenders' total expected payoff, is given by

$$
\begin{equation*}
V\left(D_{r}, D_{n}\right)=\int_{D_{n}}^{D_{n}+\frac{D_{r}}{\omega}}\left(\omega y+(1-\omega) D_{n}\right) f(y) d y+\left(D_{r}+D_{n}\right) \int_{D_{n}+\frac{D_{r}}{\omega}}^{\bar{y}} f(y) d y . \tag{2}
\end{equation*}
$$

The sovereign's ex ante welfare can be written as the total final expected surplus net of the agency costs of debt, or

$$
U_{1}=E(y)-V\left(D_{r}, D_{n}\right)-L\left(D_{n}\right)
$$

where the expected deadweight loss $L\left(D_{n}\right)$ is given by the expected value of the output lost
in a full default

$$
\begin{equation*}
L\left(D_{n}\right)=\int_{0}^{D_{n}} y f(y) d y \tag{3}
\end{equation*}
$$

The optimal debt structure thus minimizes the deadweight loss (or equivalently, the probability of a full default) subject to meeting the lenders' participation constraint,

$$
\begin{gathered}
\min _{D_{r}, D_{n}} L\left(D_{n}\right) \\
\text { subject to } V\left(D_{r}, D_{n}\right) \geq g .
\end{gathered}
$$

The deadweight loss is reduced to zero, therefore, if and only if there is no non-renegotiable debt, ${ }^{11}$

$$
D_{n}=0 .
$$

However, it may not be possible for the social planner to finance $g$ when $D_{n}=0$. For a given level of non-renegotiable debt $D_{n}$, the level of expected output that the sovereign can credibly pledge to foreign lenders is maximized when $D_{n}+D_{r} / \omega=\bar{y}$, and the maximum plegdgeable output is given by ${ }^{12}$

$$
\bar{V}\left(D_{n}\right)=\int_{D_{n}}^{\bar{y}}\left(\omega y+(1-\omega) D_{n}\right) f(y) d y .
$$

The sovereign can finance $g$ without taking the risk of a full default if $\bar{V}(0)=\omega E(y) \geq g$, that is if the bargaining power of the r-creditors is sufficiently large,

$$
\omega \geq \omega^{*} \equiv \frac{g}{E(y)}
$$

If this condition is not satisfied, the sovereign chooses the lowest level of $D_{n}$ that is consistent with the lenders' participation constraint $\bar{V}\left(D_{n}\right)=g$. The optimal level of $D_{n}$ is decreasing with $\omega$, because $\bar{V}(\cdot)$ increases with $\omega$, other things equal. An increase in the creditors' bargaining power allows the sovereign to pledge them more output, and thus to decrease its reliance on non-renegotiable debt. Conversely, in the limit case where creditors have no bargaining power $(\omega=0)$, the sovereign must finance $g$ entirely with nonrenegotiable debt. ${ }^{13}$

Our results on the first-best debt structure are summarized in the Proposition below.

[^6]Proposition 2. Assume that the sovereign debt structure is chosen by a social planner. Then there is a threshold in the creditors' bargaining power $\omega^{*}=g / E(y)$ such that:
-if $\omega<\omega^{*}$, a fraction of the sovereign debt is non-renegotiable; this fraction is decreasing with $\omega$;
-if $\omega \geq \omega^{*}$, the sovereign's debt is entirely renegotiable.
Proof. See the discussion above.

Non-renegotiable debt may have a role to play because it is a hard claim that allows the sovereign to pledge more domestic output to foreign creditors. If renegotiable debt is too soft (because creditors have too little bargaining power), some non-renegotiable debt might be required to harden the overall debt structure.

## V. Equilibrium Debt Structure

As we shall show in this section, when the sovereign borrows from multiple uncoordinated lenders the equilibrium sovereign debt structure includes an excessive level of nonrenegotiable debt. The reason is simply that for some lenders a best response to other lenders' bids is to submit a bid in the form of non-renegotiable debt, as a way of deflecting a possible selective default onto other debt issues. Moreover, the sovereign will accept these bids because they involve a lower cost of capital.

We denote by respectively $V_{r}$ and $V_{n}$, the total expected payoff of holding r-debt $D_{r}$ and ndebt $D_{n}$. Similarly, we denote by $P_{r}$ and $P_{n}$ the market prices of renegotiable and nonrenegotiable debts:

$$
\begin{gather*}
P_{r}=\frac{V_{r}}{D_{r}}=\int_{D_{n}}^{D_{n}+\frac{D_{r}}{\omega}} \frac{\omega\left(y-D_{n}\right)}{D_{r}} f(y) d y+\int_{D_{n}+\frac{D_{r}}{\omega}}^{\bar{y}} f(y) d y,  \tag{4}\\
P_{n}=\frac{V_{n}}{D_{n}}=\int_{D_{n}}^{\bar{y}} f(y) d y . \tag{5}
\end{gather*}
$$

Thus, $P_{r}$ an $P_{n}$ are the dollar values of one dollar of repayment of renegotiable and nonrenegotiable debt. One can check that non-renegotiable debt is worth more on the dollar than renegotiable debt by computing,

$$
P_{n}-P_{r}=\frac{D_{r}}{\omega} \int_{D_{n}}^{D_{n}+\frac{D_{r}}{\omega}}\left(D_{n}+\frac{D_{r}}{\omega}-y\right) f(y) d y>0
$$

The price difference reflects the effective seniority of non-renegotiable debt over renegotiable debt in the event of a selective default.

A Nash equilibrium of the lending game is a pair of debt repayments $\left(d_{r}, d_{n}\right)$ such that no lender has a strict incentive to deviate by offering a different type of debt or a different face value. It is straightforward to see that in any Nash equilibrium all lenders must just break
even: $\frac{V_{r}}{N_{r}}=\frac{V_{n}}{N_{n}}=g$. Indeed, lenders would be better off not lending if the repayments $\left(d_{r}, d_{n}\right)$ are such that $\frac{V_{r}}{N_{r}}<g$ or $\frac{V_{n}}{N_{n}}<g$. Similarly, if the repayments $\left(d_{r}, d_{n}\right)$ are such that $\frac{V_{r}}{N_{r}}>g$ (or $\left.\frac{V_{n}}{N_{n}}>g\right)$ then, since there is an excess mass of lender $(1+\varphi)$, there would be a profitable deviation for any individual lender of bidding $d_{r}-\varepsilon$, or $d_{n}-\varepsilon$ (where $\varepsilon>0$ is arbitrarily small), and thus securing a profitable loan with probability one.

Consider next a deviation where a lender switches his bid from r-debt to n-debt, resulting in infinitesimal changes $d D_{r}$ and $d D_{n}$ such that

$$
\frac{d D_{n}}{d D_{r}}=-\frac{d_{n}}{d_{r}}
$$

Given that in any Nash equilibrium the repayments $\left(d_{r}, d_{n}\right)$ are such that $P_{r} d_{r}=P_{n} d_{n}=g$, we have

$$
\begin{equation*}
P_{n} d D_{n}+P_{r} d D_{r}=0 . \tag{6}
\end{equation*}
$$

The deviation is the same as if the sovereign swapped one unit of r-debt to one unit of ndebt at market prices. The switch does not change the deviating lender's expected payoff because market prices reflect fair value, and the swap does not change the default probabilities to a first order of approximation.

A deviation from r-debt to $n$-debt is then strictly profitable if and only if the change in the sovereign's payoff $d U_{1}=-d(L+V)$ is strictly positive. Indeed, if the deviation strictly raises the sovereign's payoff then the deviating lender could share the sovereign's incremental payoff by slightly raising the face value of his debt offer.

Now, since $V=P_{r} D_{r}+P_{n} D_{n}$ and given equation (6), the decrease in the sovereign's payoff can be decomposed as follows:

$$
\begin{equation*}
d(L+V)=d L+D_{r} d P_{r}+D_{n} d P_{n} . \tag{7}
\end{equation*}
$$

Or, given that equations (3) and (5) imply $d L=-D_{n} d P_{n}$ :

$$
\begin{equation*}
d(L+V)=D_{r} d P_{r} . \tag{8}
\end{equation*}
$$

An important implication of equation (8) is that the first-order impact of a marginal change in the structure of debt on the sovereign's welfare is summarized by its effect on the price of renegotiable debt. A change in the debt structure that lowers the market price of renegotiable debt increases the sovereign's welfare without affecting the welfare of lenders. Such a change must be impossible in equilibrium, otherwise a lender could appropriate a share of the surplus through a deviating bid.

Thus, a pair of debt repayments $\left(D_{r}, D_{n}\right)$ and prices $\left(P_{r}, P_{n}\right)$ is a Nash equilibrium of the lending game if and only if: (i) equations (4) and (5) are satisfied; (ii) the market value of
debt is equal to the sovereign's financing need, $P_{r} D_{r}+P_{n} D_{n}=g$, and; (iii) any deviation from n -debt to r-debt (or vice-versa) raises the market price of r-debt: $d P_{r} \geq 0$.

We distinguish between two types of equilibria:

- interior equilibria with both types of debt ( $D_{r}>0$ and $\left.D_{n}>0\right)$;
- corner equilibria with one type of debt only ( $D_{r}=0$ or $\left.D_{n}=0\right)$.

In an interior equilibrium it must be the case that a marginal change in the structure of debt leaves the price of renegotiable debt unchanged. In a corner equilibrium, introducing the type of debt that is absent must raise the price of r-debt.

The equilibrium debt structure can then be characterized in two steps. First, we show that an interior equilibrium cannot exist.

Lemma 1. A Nash equilibrium of the lending game is such that the sovereign borrows in the form of either fully renegotiable ( $D_{n}=0$ ) or fully non-renegotiable ( $D_{r}=0$ ) debt.

Proof. See the Appendix.

Only two corner equilibria exist because the sovereign's objective function is convex in $D_{r}$ and $D_{n}$ when the prices $P_{r}$ an $P_{n}$ are held fixed. Therefore, if the sovereign could borrow arbitrary quantities of debt at given prices it would always choose a corner solution and issue only one type of debt.

Lemma 2. A corner equilibrium with non-renegotiable debt exists only if the bargaining power of creditors is such that:

$$
\omega \geq \frac{1}{2} .
$$

A corner equilibrium with renegotiable debt exists only if the bargaining power of creditors is such that:

$$
\omega \leq \bar{\omega}
$$

where $\bar{\omega}$ is larger than $\omega^{*}$ but smaller than 1.
Proof. See the Appendix.
An equilibrium with $r$-debt may not exist if a marginal deviation to $n$-debt imposes a negative externality on the outstanding r-debt, or equivalently, if it lowers the price of r-debt. As we have already argued, a deviation from $r$-debt to $n$-debt imposes a negative externality on $r$ -
debt to the extent that the sovereign is more likely to selectively default on r-debt. As the lemma highlights, however, deviations from one type of debt to the other involve more complex externalities, and there is clearly another effect at work.

To see this, it is helpful to decompose the effect of a deviation on the price of r-debt into the two effects of respectively a change in $D_{n}$ and a change in $D_{r}$. Differentiating equation (4) we obtain

$$
\begin{equation*}
d P_{r}=-\left(\frac{\omega}{D_{r}} \int_{D_{n}}^{D_{n}+\frac{D_{r}}{\omega}} f(y) d y\right) d D_{n}-\left(\int_{D_{n}}^{D_{n}+\frac{D_{r}}{\omega}} \frac{\omega\left(y-D_{n}\right)}{D_{r}^{2}} f(y) d y\right) d D_{r}, \tag{9}
\end{equation*}
$$

where $d D_{n}>0$ and $d D_{r}<0$ under a deviation from r-debt to n-debt. The first term in this equation is negative and captures the negative externality on r-debt of a marginal increase in n -debt. The second term is positive and captures the countervailing positive externality of a marginal decrease in total r-debt on the r-debt that remains outstanding.

The deviation to $n$-debt benefits the deviating lender if it decreases the price of $r$-debt, that is if the first term dominates the second term in equation (9). As we establish in the proof of Lemma 2, this is true only for $\omega$ large enough.

Why should the net externality of the deviation to $n$-debt be negative only when the bargaining power of creditors is high enough? This seems paradoxical, as creditors should worry less about a selective default if they have the upper hand in debt restructuring negotiations. The reason is that, for higher $\omega$ the sovereign's incentive to default strategically is reduced and, therefore, defaults occur only for low output realizations, $y$. As can be seen from equation (9) the second (positive) externality of a reduction in $D_{r}$ is larger for larger values of $y$. This explains why, paradoxically, an r-debt equilibrium is not sustainable for high values of $\omega$ : when creditors' bargaining power is strong the positive externality of a reduction in r-debt is small and does not outweigh the negative externality of an increase in selective defaults.

We are now ready to characterize the Nash equilibria.
Proposition 3. There are two types of Nash equilibria: a corner equilibrium with nonrenegotiable debt if and only if $\omega \in[1 / 2,1]$; and a corner equilibrium with renegotiable debt if and only if $\omega \in\left[\omega^{*}, \bar{\omega}\right]$. If $\bar{\omega}>\frac{1}{2}$, there are two Pareto-ranked corner equilibria if $\omega \in\left[\max \left(\frac{1}{2}, \omega^{*}\right), \bar{\omega}\right]$ : an inefficient equilibrium with non-renegotiable debt, and an efficient equilibrium with renegotiable debt.

Proof. The proposition follows from Lemma 2 and Proposition 2. It is possible to finance the expenditure $g$ with n-debt because of our assumption that $\max _{D_{n}} V\left(0, D_{n}\right) \geq g$, where the
lenders' expected payoff $V(\cdot, \cdot)$ is defined by (2). So the corner equilibrium with n-debt exists provided that the condition in Lemma $2 \omega \geq 1 / 2$ ) is satisfied.

Two conditions must be met for the corner equilibrium with r-debt to exist. First, $\omega$ must be high enough to allow the sovereign to pledge enough output to finance $g$, i.e., $\max _{D_{r}} V\left(D_{r}, 0\right) \geq g$. By Proposition 2 this is true if and only if $\omega \geq \omega^{*}$. Second, $\omega$ must be smaller than $\bar{\omega}$ by Lemma 2. Thus the corner equilibrium with r-debt exists if and only if $\omega \in\left[\omega^{*}, \bar{\omega}\right]$. This interval is nonempty by Lemma 2.

When $\omega \in\left[\max \left(\frac{1}{2}, \omega^{*}\right), \bar{\omega}\right]$ the conditions for the existence of an r-debt equilibrium and ndebt equilibrium are simultaneously satisfied. Finally, since the r-debt equilibrium involves no deadweight cost of default it strictly dominates the $n$-debt equilibrium. Q.E.D.

The equilibria are illustrated in Figure 1, which is constructed assuming that $y$ is uniformly distributed in $[0,2]$ and that $g=0.4$. This specification implies $\omega^{*}=0.4$ and $\widehat{\omega}=0.625 .{ }^{14}$ The figure shows how the share of non-renegotiable debt in total debt, $N_{n}=V_{n} / g$, varies with the bargaining power of the creditors, in the laissez-faire equilibrium and at the optimum.

The only values of $\omega$ for which the laissez-faire equilibrium coincide with the optimum is the interval $\left[\omega^{*}, \bar{\omega}\right]$. Recall that when lenders' ex-post bargaining power is sufficiently strong (so that $\omega>\omega^{*}$ ) the optimal debt structure is for the sovereign to issue only r-debt. As Proposition 3 highlights, there exists a range $\omega \in\left[\omega^{*}, \bar{\omega}\right]$ for which a socially efficient r-debt equilibrium exists.

If $\omega$ is outside of the rang $\left[\omega^{*}, \widehat{\omega}\right]$ the laissez-faire equilibrium is inefficient. An extreme inefficiency—market breakdown—arises when $\omega<\min \left(\frac{1}{2}, \omega^{*}\right)$. In that case, we know from Proposition 2 that the sovereign is able to raise sufficient financing $g$ only if a fraction of the debt issued is non-renegotiable. But, from Lemma 2 we also know that only an equilibrium with 100 percent $r$-debt can exists when $\omega<1 / 2$. Therefore, when $\omega<\min \left(\frac{1}{2}, \omega^{*}\right)$ the sovereign is unable to raise financing for $g$ in a laissez-faire equilibrium.

As Figure 1 suggests, and the next proposition confirms, there is generally too much n-debt issued when an equilibrium exists. This is, of course, consistent with our broad intuition that

[^7]n -debt tends to drive out r-debt.
Proposition 4 (Gresham law for sovereign debt). The socially optimal amount of n-debt is
less than or equal to the amount of n-debt issued in a Nash equilibrium for all $\omega$.

Proof. If the unique Nash equilibrium is the corner equilibrium with n-debt then there is too much n-debt in equilibrium, as, from Proposition 2, the socially optimal fraction of n-debt is strictly less than one for $\omega>0$. If the unique Nash equilibrium is the corner equilibrium with r-debt the amount of $n$-debt is equal to the socially optimal level. Q.E.D.

To summarize, the desire of individual creditors to seek protection against selective defaults by issuing n-debt tends to result in an excessive supply of $n$-debt in equilibrium and therefore an excessively high cost of default when a sovereign debt crisis arises. However, our analysis has highlighted that there may also be situations where laissez-faire can produce an efficient outcome. These are situations where the lenders' bargaining power is intermediate ( $\omega \in\left[\omega^{*}, \bar{\omega}\right]$ ). But, these are knife-edge situations: if the lenders' bargaining power is too low ( $\omega<\min \left\{\omega^{*}, \frac{1}{2}\right\}$ ) no lending equilibrium exists, and if lender bargaining power is too high $\left(\omega>\max \left\{\frac{1}{2}, \bar{\omega}\right\}\right)$ only the corner equilibrium with $n$-debt exists.

## VI. Public Policy

Given the complex effects of lenders' bargaining power in debt renegotiations on equilibrium debt structure, there is no simple welfare-improving policy intervention even in the highly simplified setting of our model. In reality, of course, the policy intervention in sovereign debt restructuring is even more complex; so much so that the debates on the bankruptcy regime for sovereigns have not resulted in any new policy initiative. The most notable new development has been a more widespread introduction of collective action clauses (CACs) in sovereign bond issues, partly as a way of preempting a more far-reaching and threatening intervention (see Gelpern and Gulati, 2007).

In terms of our model, this shift towards CACs can be interpreted as a shift towards r-debt, and to the extent that there is too much $n$-debt in equilibrium this shift might be seen as a desirable step. However, note that a purely voluntary shift from n-debt to r-debt can only be welfare improving in our model in the situation where there are multiple equilibria. In that case, it is conceivable that mild public pressure and moral suasion could serve as an equilibrium selection device and induce a switch from the $n$-debt corner equilibrium to an $r$ debt equilibrium.

In all other situations, a purely voluntary approach to CACs is unlikely to work and the implementation of some form of bankruptcy regime for sovereigns is necessary to facilitate debt renegotiations. As our analysis makes clear, however, the introduction of a bankruptcy
regime for sovereigns has both benefits and costs and may sometimes be counterproductive. Indeed, suppose that all debt becomes renegotiable under the bankruptcy regime, then the costs of this policy in our model are that when $\omega \in\left[\frac{1}{2}, \omega^{*}\right]$ the sovereign, unable to issue n debt, will be credit rationed. ${ }^{15}$ This outcome is precisely what commentators on sovereign debt restructuring like Dooley (2000) and Shleifer (2003) have been concerned about.

The bankruptcy regime is strictly beneficial only if lenders' bargaining power is sufficiently high: $\omega>\max \left(\frac{1}{2}, \omega^{*}\right)$. In that case, the sovereign is able to borrow in the form of r-debt and the elimination of $n$-debt brings about lower costs of sovereign debt crises without raising the cost of borrowing for the sovereign. Thus, the analysis in our model highlights that in the debate on the bankruptcy regime for sovereigns the advocates for reform (Krueger) as well as the critics (Dooley, Shleifer) were right, depending on the conditions. To the extent that the proponents of a bankruptcy regime had in mind a world with relatively high lender bargaining power, or envisioned a bankruptcy institution where lenders would have adequate protection, they correctly pointed to the net welfare benefits of eliminating access to n-debt for sovereign borrowers. If the critics had in mind a world with lower lender bargaining power they also correctly pointed to the risks of undermining the sovereign bond market, if the restructuring of bonds was facilitated.

Our analysis suggests two ways of reconciling these differences of opinion. One is to make sure that lenders have sufficient bargaining power in a bankruptcy regime for sovereigns. The other is to allow issuers to opt out ex-ante from the bankruptcy procedure. In other words, a policy that lets the sovereign decide whether it wants to allow for n-debt or not would guarantee that the policy intervention is always welfare improving. Thus, in our model the optimal policy is as described in the proposition below.

Proposition 5. A bankruptcy regime that makes the sovereign debt renegotiable is optimal if and only if either:

1) The regime guarantees a bargaining power to lenders such that $\omega \geq \omega^{*}$, or
2) The regime can choose to opt out of the bankruptcy regime ex ante.

Proof. See the discussion above.
We close our discussion of public policy with a note of caution, as our analysis here is restrictive in one important respect. By assuming that the amount the sovereign borrows, $g$, is fixed and known we have eliminated an important externality in sovereign debt markets: dilution of outstanding debts by new pari passu debt issues. If the sovereign cannot commit

[^8]not to borrow more than $g$ then early lenders will seek to protect themselves against this risk of dilution by issuing $n$-debt, which de facto has higher priority. We pursue the analysis of this situation in Bolton and Jeanne (2005) and show that when the sovereign cannot commit to a fixed amount of borrowing $g$ then an efficient international bankruptcy regime for sovereigns must also establish a form of seniority or absolute priority rule.

## VII. Conclusion

This paper presents a model of sovereign debt crises which, although stylized, is versatile enough to lend itself to the analysis of a number of questions that have been discussed in recent debates on the international financial architecture. The endogeneity of the debt structure implies that the normative analysis has to go beyond statements that debt workouts should be made more orderly and sovereign creditors coordinated in a crisis. These statements are correct in an ex post sense, but from an ex ante perspective debt structures with non-renegotiable debt arise for a reason.

At the same time, our analysis does not support a Panglossian view that sovereign debt contracts are efficient ex ante and that there is no scope for welfare-improving reforms. We do find that sovereign debt might be excessively difficult to restructure under laissez-faire (even from an ex ante point of view), and that public intervention is warranted.

This model abstracted from a number of issues that may be quite relevant in the real world. One such issue is debt maturity. Short-term debt is another way of deflecting selective defaults. However, short-term debt could make sovereigns excessively vulnerable to debt rollover crises (Jeanne, 2004). This issue is addressed in Bolton and Jeanne (2005) and requires a different form of intervention than the simple facilitation of debt renegotiations.

Our analysis could also be extended to take other agency problems than those between debtors and creditors into consideration, in particular political agency problems between citizens and their governments. In our model it is unambiguously optimal to relax the credit constraints in the international debt market because governments are assumed to be benevolent. The welfare analysis could be very different if decisions are taken by selfinterested policymakers who do not maximize domestic welfare. Rationing the debt granted to policymakers, then, could conceivably increase the welfare of their citizens.

## Proof of Lemma 1

Consider an interior equilibrium ( $D_{r}, D_{n}$ ) with $D_{r}>0$ and $D_{n}>0$. In such an equilibrium, a marginal change in the debt structure such that $P_{n} d D_{n}+P_{r} d D_{r}=0$ must have a zero firstorder effect on the sovereign's loss $L+V$.

We show that the second-order effect,

$$
\begin{equation*}
d^{2}(L+V)=\frac{\partial^{2}(L+V)}{\partial D_{r}^{2}} d D_{r}^{2}+2 \frac{\partial^{2}(L+V)}{\partial D_{r} \partial D_{n}} d D_{r} d D_{n}+\frac{\partial^{2}(L+V)}{\partial D_{n}^{2}} d D_{n}^{2} \tag{10}
\end{equation*}
$$

is negative, implying that the sovereign benefits from any marginal change in the debt structure such that $P_{n} d D_{n}+P_{r} d D_{r}=0$.
By (2) and (3) $L+V$ can be written as:

$$
L+V=\int_{0}^{D_{n}} y f(y) d y+\int_{D_{n}}^{D_{n}+\frac{D_{r}}{\omega}}\left(\omega y+(1-\omega) D_{n}\right) f(y) d y+\left(D_{r}+D_{n}\right) \int_{D_{n}+\frac{D_{r}}{\omega}}^{\bar{y}} f(y) d y .
$$

The function under the integral signs is continuous at the thresholds $D_{n}$ and $D_{n}+\frac{D_{r}}{\omega}$. Therefore, the derivative of $(L+V)$ with respect to $D_{r}$ and $D_{n}$ can be obtained by differentiating the terms in $D_{r}$ and $D_{n}$ under the integral sign, which gives

$$
\begin{aligned}
\frac{\partial(L+V)}{\partial D_{n}} & =(1-\omega) \int_{D_{n}}^{D_{n}+D_{r} / \omega} f(y) d y+\int_{D_{n}+D_{r} / \omega}^{\bar{y}} f(y) d y, \\
\frac{\partial(L+V)}{\partial D_{r}} & =\int_{D_{n}+D_{r} / \omega}^{\bar{y}} f(y) d y, \\
\frac{\partial^{2}(L+V)}{\partial D_{n}^{2}} & =-\omega f\left(D_{n}+\frac{D_{r}}{\omega}\right)-(1-\omega) f\left(D_{n}\right), \\
\frac{\partial^{2}(L+V)}{\partial D_{n} \partial D_{r}} & =-f\left(D_{n}+\frac{D_{r}}{\omega}\right), \\
\frac{\partial^{2}(L+V)}{\partial D_{r}^{2}} & =-\frac{1}{\omega} f\left(D_{n}+\frac{D_{r}}{\omega}\right) .
\end{aligned}
$$

Substituting these expressions into (10) in turn gives,

$$
d^{2}(L+V)=-(1-\omega) f\left(D_{n}\right) d D_{n}^{2}-\frac{1}{\omega} f\left(D_{n}+\frac{D_{r}}{\omega}\right)\left(\omega d D_{n}+d D_{r}\right)^{2}<0
$$

## Proof of Lemma 2

We shall use the criterion stated in the text: in a corner equilibrium, introducing the type of debt that is absent must raise the price of $r$-debt. Consider first the corner equilibrium with $r$ -
debt only. Let $P_{r}$ and $D_{r}$ respectively denote the price and quantity of r-debt in this equilibrium. Now replace an infinitesimal amount of r-debt, $d D_{r}$, by an infinitesimal amount of n-debt, $d D_{n}$. The price of the new n-debt is equal to 1 to a first-order of approximation (since the probability of a full default is infinitesimal). Differentiating (4) and using (6), $P_{n}=1$, and $P_{r} D_{r}=g$, we have

$$
\begin{aligned}
d P_{r} & =-\frac{\omega d D_{n}}{D_{r}} \int_{0}^{\frac{D_{r}}{\omega}} f(y) d y-\frac{\omega d D_{r}}{D_{r}^{2}} \int_{0}^{\frac{D_{r}}{\omega}} y f(y) d y \\
& =-\frac{\omega d D_{n}}{g D_{r}} \int_{0}^{\frac{D_{r}}{\omega}}(g-y) f(y) d y .
\end{aligned}
$$

Now consider the variations of the function $h: \omega \mapsto \int_{0}^{\frac{D_{t}}{\omega}}(g-y) f(y) d y$. We show that there exists a threshold $\bar{\omega} \in\left[\omega^{*}, 1\right]$ such that $h(\omega)$ is negative if and only i $\omega \leq \bar{\omega}$. Since $d P_{r}$ must be positive in a corner equilibrium with $r$-debt, it then follows that such an equilibrium exists if and only if $\omega \leq \bar{\omega}$, as stated in the Proposition.

To establish that a threshold $\bar{\omega} \in\left[\omega^{*}, 1\right]$ exists we must show first that $h(\cdot)$ is an increasing function in $\omega$. The equilibrium level of $D_{r}$ is a function of $\omega$ implicitly defined by the lenders' budget constraint,

$$
\begin{equation*}
g=\int_{0}^{\frac{D_{r}}{\omega}} \omega y f(y) d y+D_{r} \int_{\frac{D_{r}}{\omega}}^{\bar{y}} f(y) d y . \tag{11}
\end{equation*}
$$

Dividing this equation by $\omega$ gives

$$
\frac{g}{\omega}=\int_{0}^{m(\omega)} y f(y) d y+m(\omega) \int_{m(\omega)}^{\bar{y}} f(y) d y,
$$

where $m(\omega)=\frac{D_{r}}{\omega}$. Differentiating this expression with respect to $\omega$ we see that $m^{\prime}(\omega)<0$. Using $g<D_{r} \leq \frac{D_{r}}{\omega}=m(\omega)$, it follows that

$$
h^{\prime}(\omega)=m^{\prime}(\omega)(g-m(\omega))>0 .
$$

The existence of $\hat{\omega} \in\left[\omega^{*}, 1\right]$ then follows from the facts that $h\left(\omega^{*}\right)<0$ and $h(1)>0$. To establish the first inequality note that since $m\left(\omega^{*}\right)=\bar{y}$, we have $h\left(\omega^{*}\right)=g-E(y)<0$. The second inequality follows from equation (11), which for $\omega=1$ reduces to:

$$
h(1)=\int_{0}^{D_{r}}(g-y) f(y) d y=\left(D_{r}-g\right) \int_{D_{r}}^{\bar{y}} f(y) d y>0 .
$$

Consider next the corner equilibrium with n-debt only. Let $P_{n}$ and $D_{n}$ respectively denote the price and quantity of $n$-debt in this equilibrium. Again replace an infinitesimal amount of n-debt, $d D_{n}$, by an infinitesimal amount of r-debt, $d D_{r}$. The price of the new r-debt is equal to $P_{n}$ to a first-order of approximation (since selective default occurs with an infinitesimal probability). Differentiating (4) and using (6) and $P_{n}=P_{r}$, we have

$$
\frac{d P_{r}}{d D_{r}}=-\omega\left(\frac{1}{d D_{r}^{2}} \int_{D_{n}}^{D_{n}+\frac{d D_{r}}{\omega}}\left(y-D_{n}\right) f(y) d y-\frac{1}{d D_{r}} \int_{D_{n}}^{D_{n}+\frac{d D_{r}}{\omega}} f(y) d y\right)
$$

Using L'Hospital rule we can show that

$$
\begin{gathered}
\lim _{d D_{r} \rightarrow 0} \frac{1}{d D_{r}} \int_{D_{n}}^{D_{n}+\frac{d D_{r}}{\omega}} f(y) d y=\frac{f\left(D_{n}\right)}{\omega} \text { and } \\
\lim _{d D_{r} \rightarrow 0} \frac{1}{d D_{r}^{2}} \int_{D_{n}}^{D_{n}+\frac{d D_{r}}{\omega}}\left(y-D_{n}\right) f(y) d y=\frac{f\left(D_{n}\right)}{2 \omega^{2}},
\end{gathered}
$$

from which it follows that

$$
\lim _{d D_{r} \rightarrow 0} \frac{d P_{r}}{d D_{r}}=-f\left(D_{n}\right)\left(\frac{1}{2 \omega}-1\right) .
$$

This limit is positive (as it must be if $n$-debt is a corner equilibrium) if and only if $\omega>1 / 2$.

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Figure 1. Share of non-renegotiable debt and bargaining power of creditors



The figure is based on the following specification of the model: $y$ is uniformly distributed in $[0,2]$ and $g=0.4$.


[^0]:    ${ }^{1}$ See Rogoff and Zettelmeyer (2002) for a history and overview of the different proposals.

[^1]:    ${ }^{2}$ The idea that under limited enforcement it may be desirable to create a debt structure that is difficult to renegotiate is, of course, a familiar theme in corporate finance. See, for example Hart and Moore (1995), Dewatripont and Maskin (1995), Bolton and Scharfstein (1996), Diamond and Rajan (2001) and Diamond (2004).

[^2]:    ${ }^{3}$ The contractual approach advocated by the official sector is outlined in G-10 (1996) and G-22 (1998).
    ${ }^{4}$ Collective action clauses facilitate bond restructurings by lowering the threshold for agreement to a restructuring by bondholders from unanimity to a 75 percent super-majority rule.

[^3]:    ${ }^{5}$ Michael Peterson, "A crash course in default," Euromoney (October 1999), pp. 47-50.
    ${ }^{6}$ Debt restructuring was difficult but not impossible, thanks to a creative use of exit consent clauses (Buchheit and Gulati, 2000), leading Roubini and Setser (2004) to conclude that the lack of creditor coordination was overstated as an impediment to debt restructuring. However, the expectation that bonded debt would be difficult to restructure seems to have played a significant role in shaping the equilibrium structure of sovereign debt in the 1990s.

[^4]:    ${ }^{7}$ See Bolton and Jeanne (2005) for a more general model where $g$ is optimally determined by the sovereign.

[^5]:    ${ }^{8}$ It is generally assumed in the literature that the cost of defaulting is the same whether the sovereign defaults in full or whether it repays part of its debt. This is a somewhat extreme assumption. One might want to consider the more general default cost function $\gamma(s) y$, where $\gamma(s)$ is increasing in the repayment shortfall $s$ from zero to a maximum value, $\bar{\gamma}<1$. Our analysis would be virtually unchanged if we allowed for this more general default cost function.
    ${ }^{9}$ Another approach views the cost of default as a loss of reputation (e.g., Eaton and Gersowitz, 1981). See Bolton and Jeanne (2005) for a model of sovereign debt restructuring that includes both types of cost.
    ${ }^{10}$ The inability to renegotiate the debt ex-post may be to the detriment of bondholders' collective interests. Even so, because of a free-rider problem - as in Diamond and Rajan (2001) or Jeanne (2004)- widely dispersed debts will not be renegotiable ex-post. For example, individual litigating creditors could hope to seize some collateral, but if they litigate in an uncoordinated way, these creditors might impose an output cost on the country that is much larger than the value of collateral that they can seize collectively. Similarly, the bondholders may be unable to accept a voluntary decentralized debt exchange or repurchase, even an efficient one, because of free-riding by holdouts (Bulow and Rogoff, 1991).

[^6]:    ${ }^{11}$ This result is due to our assumption that output realizations can be arbitrarily small. If the distribution of output had a strictly positive lower bound $\underline{y}$, then a zero deadweight loss would only require that $D_{n} \leq \underline{y}$. Our results can be generalized to this case without difficulty.
    ${ }^{12}$ Differentiating (2) shows that $V$ is strictly increasing with $D_{r}$ if $D_{n}+D_{r} / \omega<\bar{y}$.
    ${ }^{13}$ We assume that this is possible because $\max _{D_{n}} V\left(0, D_{n}\right) \geq g$.

[^7]:    ${ }^{14}$ Closed-form solutions for the equilibrium can be derived in the case of a uniform distribution. The details are available upon request to the authors.

[^8]:    ${ }^{15}$ The sovereign is rationed only if it was able to borrow with n-debt and not with r-debt under laissez-faire, that is if $\omega \geq 1 / 2$ and $\omega \leq \omega^{*}$.

